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The adjacency spectra of some families of minimally connected prime graphs. (English)

Zbl 1535.05131
Involve 17, No. 1, 107-120 (2024).

Given a finite group G, its prime graph Γ_G is the graph whose vertices are the prime divisors of the order of G, and such that $\{p, q\}$ is an edge if and only if there is an element of order pq in G. In [*A. Gruber* et al., J. Algebra 442, 397–422 (2015; Zbl 1331.20029)], it is shown that a graph Γ is isomorphic to the prime graph of a finite solvable group if and only if its complement $\overline{\Gamma}$ is triangle-free and 3-colorable.

Let $\mathcal{H}$ be the family of all prime graphs of finite solvable groups. A minimal prime graph (MPG) is a nontrivial connected graph $\Gamma \in \mathcal{H}$ such that, for each edge $e \in E(\Gamma)$, $\Gamma - e \notin \mathcal{H}$. A minimally connected prime graph (MCPG) is a nontrivial connected graph $\Gamma \in \mathcal{H}$ such that, for each edge $e \in E(\Gamma)$, $\Gamma - e \notin \mathcal{H}$ or $\Gamma - e$ is not connected. Let $\mathcal{G}$ be the family of all MPGs and $\widehat{\mathcal{G}}$ be the family of all MCPGs.

In this paper, the authors study some classes of MCPGs from a linear algebraic perspective. The complete bridge graph $B_{m,n}$ is the graph obtained by adding an edge between a copy of K_m and a copy of K_n. A reseminant graph is a graph obtained from the cycle C_5 by duplicating vertices. Let $\mathcal{R}$ denote the family of reseminant graphs, and $\widetilde{\mathcal{R}}$ denote the subfamily of those graphs obtained by repeated duplication of a single vertex. In [*C. Florez* et al., "Minimal Prime Graphs of Solvable Groups", Preprint, arXiv:2011.10403] it is shown that $\widehat{\mathcal{G}} \backslash \mathcal{G}$ consists of the graphs $B_{m,n}$ where $m \geq n > 1$ or $(m, n) \in \{(1, 1), (2, 1)\}$, and that $\mathcal{R} \subsetneq \mathcal{G}$.

The main results in this paper are (a) the computations of the characteristic polynomials of $B_{m,n}$ (for $m + n > 4$) and the graphs in $\mathcal{R}$; (b) the computations of the spectra of $B_{m,m-1}$ (for $m > 2$) and the graphs in $\widetilde{\mathcal{R}}$; (c) the following characterization of the subfamily $\widetilde{\mathcal{R}}$: if $\Gamma \in \mathcal{R}$, then $\Gamma \in \widetilde{\mathcal{R}}$ if and only if Γ has at least two adjacent vertices of degree 2.

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MSC:

05C25 Graphs and abstract algebra (groups, rings, fields, etc.)
15A18 Eigenvalues, singular values, and eigenvectors
20D10 Finite solvable groups, theory of formations, Schunck classes, Fitting classes, π-length, ranks

Keywords:

prime graphs; adjacency matrix; spectral graph theory

Full Text: DOI arXiv

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